

METHOD OF DETERMINING THE THERMAL DEFORMATIONS OF  
ASTRONOMICAL MIRRORS

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Translation of "Metod opredeleniya termicheskikh deformatsiy astron-  
omicheskikh zerkal", Pulkovo, Glavnaya Astronomicheskaya observatoriia,  
Izvestiia, No. 186, 1971, pp. 189-191

{NASA-TM-77941} METHOD OF DETERMINING THE  
THERMAL DEFORMATIONS OF ASTRONOMICAL MIRRORS  
(National Aeronautics and Space  
Administration) 9 p HC A02/MF A01 CSCL 03A

N86-22457

Unclas  
G3/89 05699



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
WASHINGTON, D.C. 20546 DECEMBER 1985

## STANDARD TITLE PAGE

1. Report No. TM-77941	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle METHOD OF DETERMINING THE THERMAL DEFORMATIONS OF ASTRONOMICAL MIRRORS		5. Report Date December 1985	6. Performing Organization Code
		8. Performing Organization Report No.	10. Work Unit No.
7. Author(s) Ye. G. Khablo-Grossval'd		11. Contract or Grant No. NASW- 4005	
		12. Type of Report and Period Covered Translation	
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063		14. Sponsoring Agency Code	
12. Sponsoring Agency Name and Address National Aeronautics and Space Admini- stration, Washington, D.C. 20546			
15. Supplementary Notes  Translation of "Metod opredeleniya termicheskikh defor- matsiy astronomicheskikh zerkal", Pulkovo, Glavnaya observatoriia, Izvestiia, No. 186, 1971, pp. 189-191 (A72-37969)			
16. Abstract  The temperature conditions of the operation of astronomical mirrors are considered. The formulas for determining the temperature fields for some forms of mirrors are given. The case of eccentric irradiation of a part of the working surface of a mirror is discussed. The method of modeling of the thermal deformations in the case of complex tempera- ture fields is described.			
17. Key Words (Selected by Author(s))		18. Distribution Statement  Unlimited-Unclassified	
19. Security Classif. (of this report)  Unclassified	20. Security Classif. (of this page)  Unclassified	21. No. of Pages  7	22.

# METHOD OF DETERMINING THE THERMAL DEFORMATIONS OF ASTRONOMICAL MIRRORS

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Given herein are the calculations of temperature fields of astronomical mirrors, frequently encountered in astronomical instrument building, and the thermal deformations which correspond to them. A procedure is described for modeling the thermal deformations in the case of complex fields.

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Thermal deformations occur most frequently because of temperature gradients along the thickness and diameter of mirrors. In this case, a change is observed in the curvature of the surface. This may be traced most graphically in solar telescopes and in extraterrestrial astronomy, with the presence of large radiation outputs. A considerable coefficient of linear expansion of the material of the mirror evokes deformations which may be eliminated by refocusing only in isolated cases. And, finally, there is the edge effect, or the Reech effect, observed in mirrors with a low thermal conductivity and taking place because of the non-uniformity of the temperature field in the mass and along the surface. We know of such measures for combating thermal deformations as decreasing of the thickness of the mirror, selection of a material with a small coefficient of linear expansion, separation of the mirrors, their protection with thermal insulation, forced ventilation, and careful selection of the construction site of the telescope.

The determination of the thermal deformations is divided into two stages: 1) determination of the temperature field and 2) determination of the temperature deformations which correspond to it.

The task of the first stage may be formulated as follows. There is a thick circular plate  $2a$  in diameter and  $2h$  thick. It is located in a temperature field which is symmetrical relative to the

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\*Numbers in the margin indicate pagination in the foreign text.

z-axis. The boundary value conditions are given. Assuming the field to be stationary and not having internal heat sources, we will obtain the Laplace equation in the general form from the differential equation of thermal conductivity, which will have the following form, in operator notation, in a cylindrical coordinate system with regard for the symmetry relative to the z-axis:

$$\left(r \frac{\partial^2}{\partial r^2} + \frac{\partial}{\partial r} + r \frac{\partial^2}{\partial z^2}\right) T = 0.$$

By solving this equation according to the Fourier method, we will arrive at the Bessel equation. By applying restrictions, characteristic for our problem, to its general solution, we will obtain the solution in the form

$$T = J_0(kr) [Ae^{kz} + Be^{-kz}].$$

The coefficients A, B, k are determined from the boundary value conditions. By examining this equation for a series of partial cases, we obtain:

1. For the case of mirrors with a spherical surface:

$$T = J_0(kr) \left[ \frac{T_3 \operatorname{sh} k(z+h) - T_1 \operatorname{sh} k(z-l)}{\operatorname{sh}(km)} \right], \quad l = h - \frac{a^2}{2R}.$$

The constant k is determined graphically for each concrete problem according to the intersection of the curves which correspond to two functions:  $J_0(ka)$  and  $T_2 \operatorname{sh}(km) / T_1 \operatorname{sh}(kl) + T_3 \operatorname{sh}(kh)$ , where  $m=h+l$ .

2. For the case of a mirror with a flat surface:

$$T = \frac{1}{2} J_0(kr) \left[ \frac{\operatorname{ch} kz}{\operatorname{ch} kh} (T_3 + T_1) + \frac{\operatorname{sh} kz}{\operatorname{sh} kh} (T_3 - T_1) \right].$$

we will obtain k graphically from the expression  $2T_2 / T_1 + T_3 = J_0(ka) / \operatorname{ch}(kh)$ , and we will make it more specific by the method of iterations.

3. For a mirror with a spherical surface and a cylindrical central aperture: /190

$$T = \frac{J_0(kr)}{J_0(k\rho)} \left[ \frac{T_3 \operatorname{sh} k(h+z) - T_1 \operatorname{sh} k(h-s-z)}{\operatorname{sh} k(2h-s)} \right],$$

where  $S = a^2 - r^2 / 2R$ ,  $T_1$ ,  $T_2$  and  $T_3$  are the temperature on the spherical, side and lower surfaces, respectively. In a partial case, namely with the presence of a gradient by thickness, and knowing the temperature

field of the mirror, one may calculate its deflection indicator according to the formula [1]

$$\Delta x = \frac{D^2 \alpha \Delta t}{8l},$$

where  $D$  is the diameter of the mirror,  $l$  is the thickness of the mirror,  $\Delta t$  is the temperature gradient, and  $\alpha$  is the coefficient of linear expansion.

In addition to this problem, we must touch on other conditions of operation of the mirrors. In the general form, one may formulate this problem in the following manner. We are given a circular plate, which is located in a temperature field. The temperature  $T_1$  is given in its upper plane, in the area of a circle of radius  $r$ . The

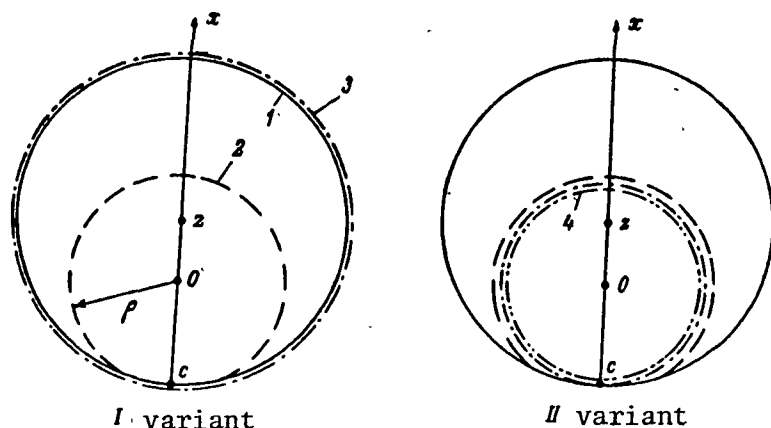


Fig. 1. Calculation of deformation at point C.

1—actual mirror; 2—actual illumination;  
3—calculated illumination; 4—calculated mirror.

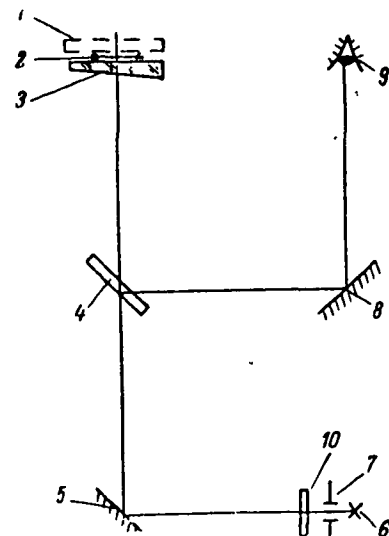


Fig. 2. Schematic of interferometer.

center of the circle is shifted relative to the center of the plate by  $\Delta$ . Outside of the circle, the temperature is equal to  $T_2$ . A similar problem, only with a circle which is symmetrical relative to the center of the plate, has been solved by Novatskiy [2].

For an approximated solution, we will reduce our problem to that solved by the replacement of the eccentric region by a region

with a center on the z-axis, but a variable radius:  $\rho = \Delta \cos \beta + \sqrt{2^2 - \Delta^2 \sin^2 \beta}$ , where  $\beta$  is the angle between the x-axis and the current radius (Fig. 1). The deformations of the mirror will be expressed as

$$u_z = (1 + \nu) \alpha_t \rho T_0 \sum_{n=1}^{\infty} \frac{J_1(\alpha_n \rho) J_0(\alpha_n^2) \lg h(\alpha_n h)}{(\alpha_n R)^2 J_1^2(\alpha_n R)} - 2 \alpha_t \rho \nu T_0 \sum_{n=1}^{\infty} \frac{J_1(\alpha_n \rho) \lg h(\alpha_n h)}{(\alpha_n R)^3 J_1(\alpha_n R)},$$

$$T_0 = T_1 - T_2 = \text{const.}$$

where  $\nu$  is the Poisson coefficient, and the remaining designations are as before. This very same problem may be solved by another method: assuming a region of illumination of constant radius, and the diameter of the mirror as variable. By examining both variants of the solution, we will find the one that is most appropriate for our problem.

If the temperature field is complex, then the deformations which occur in it are determined by experimental means. Work in this direction has also been done in two stages: modeling of the temperature field and measurement of the deformations. Devitrified glass, quartz and Pyrex mirrors have been studied. Monitoring of the thermal state of the mirror was carried out at six points along the rear and working surfaces: in the center, along the edge, and in the middle of the radius. The temperature in them was determined by highly-sensitive type MT-54 heat gauges, manufactured by the workshops of the Leningrad Agrophysical Institute. The gauges operate reliably in the temperature range from  $-70$  to  $+150^\circ \text{C}$ . The stability of the nominal resistance is  $0.3\%$  in a static mode. The gauges, calibrated on a special unit, are introduced into the schematic of the EPP-09, and the scale of the latter is graduated for each gauge separately, in order that continuous monitoring be carried out at all six points of the mirror being studied. The required temperature field was created in the mirror by the contact method. A vessel was constructed, into which water was poured and a coil spring placed. The vessel was placed in contact with the mirror through a small steel pivot, which possesses great thermal conductivity. Between the remaining surface of the bottom of the vessel and the mirror was an air gap, the optimal size of which was selected by experimental means. It was possible to obtain the required shape of the contact surface of the vessel, proceeding from the given temperature field.

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However, a curve is obtained which is difficult to implement. By heating the mirror and tracing the temperature data according to the EPP-09, we established the onset of the thermal state which is closest to the given state. Then, the mirror was mounted on an interferometer, and the interference picture was taken. The schematic of the interferometer, specially designed and constructed for this study, is given in Figure 2. Utilized as an illumination source in it was an SMR-1 mercury lamp, and the line with a wavelength  $\lambda=5461 \text{ \AA}$  was picked out of the entire spectrum of the lamp using a green filter. The light of the lamp, reflected by the flat mirror 5, is

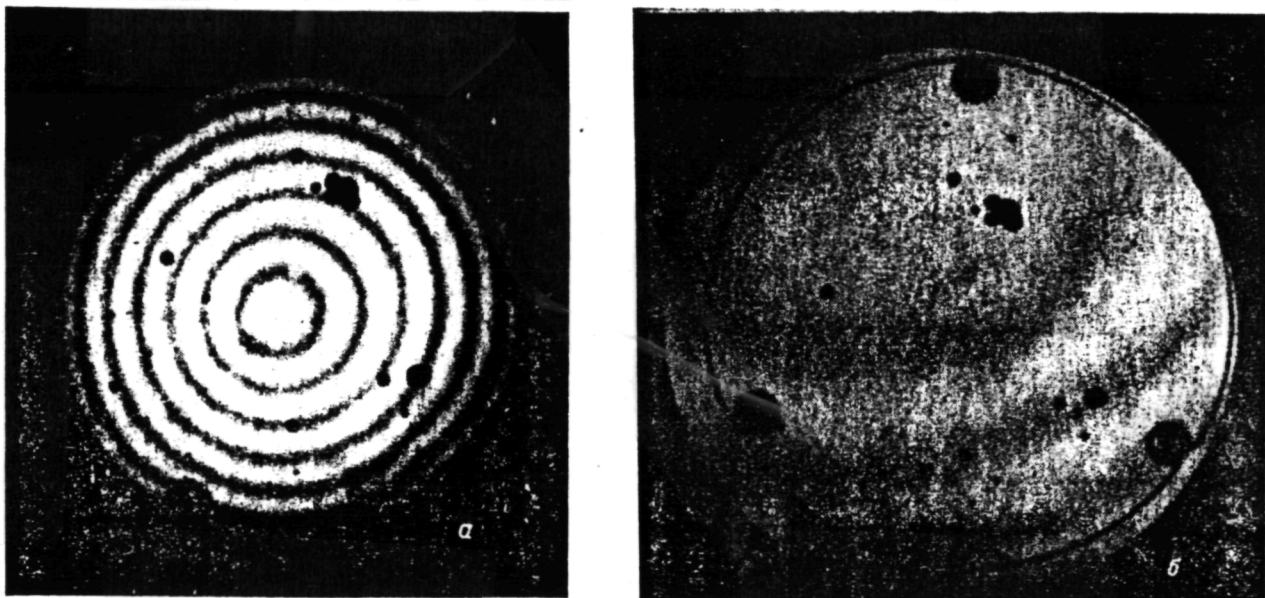


Fig. 3. Picture of interference zones for Pyrex (a) and quartz (b).

sent to the interferometer proper, which consists of a standard wedge 3 and the quasi-flat surface being tested 1. The beam, having been interferometrically processed, is reflected using a planar-parallel plate 4 in the direction of the flat mirror 8, and from it, to the eyes of the observer or the "Zenit" type photographic apparatus. The planar-parallel plate 4 has two-layer chemical coating of the lower surface, and dielectric semitransparent coating of the upper surface. The coating precludes doubling of the image, and the dielectric coating increases the brightness of the picture. The photographing of the interference picture was carried out with an exposure of about 1 second. Film from the firm "ORWO" was utilized

with a sensitivity of 250 units, according to the G.O.S.T.. The measurement of the obtained rings was carried out on a universal measuring microscope. According to the data of the measurements, we calculated the deformations of the mirrors, with regard for the initial processing of the surface. Simultaneously, the films were processed for verification on the MOL microphotometer. Experiments were carried out with mirrors 125 mm in diameter and 16 mm thick. As an example, pictures are given of the interference zones of Pyrex (a) and quartz (b), for temperature gradients of 10 and 36° C, respectively (Fig. 3).



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